Solutions of Mock JEE Advanced – 2 | Paper - 1 | JEE 2024

PHYSICS

Section – 1

1.(BCD) 1 in the subscript stands for part of circuit containing capacitor C & resistance $2R$.

2 in the subscript stands for part of circuit containing capacitor $2C$ & resistance R.

Charges on capacitors in steady state is as follows.

$$\therefore \quad (q_1)_f = CE$$

$$(q_2)_f = 2CE$$

$$\therefore \quad \text{Ratio} \left(\frac{(q_1)_f}{(q_2)_f}\right) = \frac{1}{2}$$

Further $\tau = (2C)R = C(2R) = 2CR$

Charges on capacitors at any time (t)

$$q_1 = CE\left(1 - e^{-t/2CR}\right)$$

$$q_2 = 2CE\left(1 - e^{-t/2CR}\right)$$

$$\frac{q_1}{q_2} = \frac{1}{2}$$

Current in capacitor at any time (t)

$$i_1 = \frac{E}{2R}\left(e^{-t/2CR}\right)$$

$$i_2 = \frac{E}{R}\left(e^{-t/2CR}\right)$$

$$\frac{i_1}{i_2} = \frac{1}{2}$$

2.(A, C) $E_{in} = \frac{\mu_0}{2\pi} Iv \ln 2$

$\therefore$ Charge on capacitor at time t, $Q = \frac{C\mu_0 IV}{2\pi} \ln 2\left(1 - e^{-t/RC}\right)$

Current $I = \frac{dQ}{dt} = \frac{\mu_0 Iv \ln 2}{2\pi R} e^{-t/RC}$

3.(CD) For Process ab

$\rho = \text{constant}$

$V = \text{constant}$

$\therefore \quad W_{ab} = 0$

$Q_{ab} = \Delta U$

ΔU is positive, as U is increasing.

Hence, Q_{ab} is also positive.

For process bc

$\rho \propto U \Rightarrow \therefore \frac{1}{V} \propto T$

ρ is decreasing, So V is increasing. Hence, work done is positive. (i.e. $W_{bc} > 0$)

Further, $\frac{1}{V} \propto T$ ...(i)

$T \propto PV$ ...(ii)

From the above two relations, we get

$\therefore \quad PV^2 = \text{constant}$

In the process, $PV^x = \text{constant}$, Molar heat capacity is given by

$$C = C_v + \frac{R}{1-x}$$

Here, $x = 2$

$\therefore \quad C = C_v - R$

For any of the gas, $C_v \neq R$

$\therefore \quad C \neq 0 \quad \therefore \quad Q_{bc} = nC\Delta T \neq 0$ as $\Delta U \neq 0$ and $\Delta T \neq 0$

For process ca

ρ is increasing. Hence, V is decreasing. So, work done is negative. (i.e. $W_{ca} < 0$)

4.(AC) $\mu = \dfrac{\sin\left(\frac{A+\gamma}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \cos\left(\frac{\gamma}{2}\right) + \cot\left(\frac{A}{2}\right)\sin\left(\frac{\gamma}{2}\right)$

$$\mu = \frac{\sin\left(\frac{A+\gamma}{2}\right)}{\sin\left(\frac{A}{2}\right)} = \cos\left(\frac{\gamma}{2}\right) + \cot\left(\frac{A}{2}\right)\sin\left(\frac{\gamma}{2}\right) = \cos\left(\frac{\gamma}{2}\right) + \sin\left(\frac{\gamma}{2}\right) \qquad \ldots\text{(i)}$$

Also $\cos^2\left(\frac{\gamma}{2}\right) + \sin^2\left(\frac{\gamma}{2}\right) = 1$ …(ii)

Solving equation (i) and (ii), we get $\sin\gamma = \mu^2 - 1$

$i = 90°$

also $\beta = i + e - A$

$\beta = e$

Also $\sin\beta = \mu\cos C$ (c = critical angle)

$\sin C = \dfrac{1}{\mu}$

$\cos C = \dfrac{\sqrt{\mu^2 - 1}}{\mu}$

5.(ABD)

$E = \dfrac{k\,q_{in}}{r^2}$ $\left(\text{Here}, k = \dfrac{1}{4\pi\varepsilon_0}\right)$

$\therefore$ $E_A = E_c = 0$

But, $E_B \neq 0$

$V = \dfrac{kq}{R}$ $(r \leq R)$

$V = \dfrac{kq}{r}$

6.(BC) At $t = 4T$

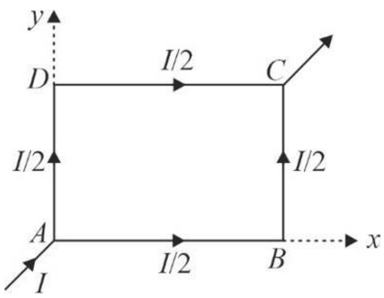
Number of half-lives of first $n_1 = 4$ and number of half-lives of second $n_2 = 2$

$$\frac{N_1}{N_2} = x = \frac{N_0(1/2)^4}{N_0(1/2)^2} = \frac{1}{4}$$

$$y = \frac{R_1}{R_2} = \frac{\lambda_1 N_0(1/2)^4}{\lambda_2 N_0(1/2)^2} = \frac{\lambda_1}{4\lambda_2} = \frac{T_2}{4T_1} = \frac{2T}{4T} = \frac{1}{2}$$

Section – 2

7.(A)



(I) When $\vec{B} = B_0\hat{i}$

Force on wire $AB\left(\vec{F}_{AB}\right) =$ Force on wire $CD(\vec{F}_{CD}) = \vec{0}$ (as $\vec{l}$ & $\vec{B}$ are in same direction)

Force on wire $BC\left(\vec{F}_{BC}\right) = \left(\frac{Il}{2}\right)\hat{j} \times \left(B_0\right)\hat{i}$

$$\vec{F}_{BC} = -\frac{IlB_0}{2}\hat{k}$$

Force on wire $AD\left(\vec{F}_{AD}\right) = \left(\frac{Il}{2}\right)\hat{j} \times \left(B_0\right)\hat{i}$

$$\vec{F}_{AD} = -\frac{IlB_0}{2}\hat{k}$$

$\therefore$ Net force on wire loop $\left(\vec{F}_{Net}\right) = \vec{F}_{AB} + \vec{F}_{BC} + \vec{F}_{CD} + \vec{F}_{AD}$

$\Rightarrow$ $\vec{F}_{Net} = -\left(ilB_0\right)\hat{k}$

$\therefore$ Magnitude of net force on the loop $ABCD$ is $\left(B_0 Il\right) N$

(II) When $\vec{B} = B_0\hat{j}$

Force on wire $AD\left(\vec{F}_{AD}\right) =$ Force on wire $BC\left(\vec{F}_{BC}\right) = \vec{0}$ (as $\vec{l}$ & $\vec{B}$ are in same direction)

Force on wire $AB\left(\vec{F}_{AB}\right) = \left(\frac{Il}{2}\right)\hat{i} \times \left(B_0\right)\hat{j}$

$$\vec{F}_{AB} = \left(\frac{IlB_0}{2}\right)\hat{k}$$

Force on wire $CD\left(\vec{F}_{CD}\right)=\left(\frac{Il}{2}\right)\hat{i}\times\left(B_0\right)\hat{j}$

$$\vec{F}_{CD}=\left(\frac{IlB_0}{2}\right)\hat{k}$$

$\therefore$ Net force on loop $ABCD$ $\left(\vec{F}_{Net}\right)=\vec{F}_{AB}+\vec{F}_{BC}+\vec{F}_{CD}+\vec{F}_{AD}$

$$\vec{F}_{Net}=\left(IlB_0\right)\hat{k}$$

So magnitude of net force on wire loop is $\left(IlB_0\right)N$

(III) When $\vec{B}=B_0\left(\hat{i}+\hat{j}\right)$

$\therefore$ Net force in this case will be vector sum of first two cases that we have just solved.

$\therefore$ $\vec{F}_{net}=ilB_0\hat{k}-ilB_0\hat{k}=\vec{0}$

So magnitude of net force on wire loop is zero in this case (i.e. $F_{Net}=0N$)

(IV) When $\vec{B}=B_0\hat{k}$

$\therefore$ $\vec{F}_{AB}=\left(\frac{Il}{2}\right)\hat{i}\times\left(B_0\right)\hat{k}$

$$\vec{F}_{AB}=-\frac{IlB_0}{2}\hat{j}$$

$$\vec{F}_{BC}=\left(\frac{Il}{2}\right)\hat{j}\times\left(B_0\right)\hat{k}$$

$$\vec{F}_{BC}=\frac{IlB_0}{2}\hat{i}$$

$$\vec{F}_{CD}=\left(\frac{Il}{2}\right)\hat{i}\times\left(B_0\right)\hat{k}$$

$$\vec{F}_{CD}=\frac{-IlB_0}{2}\hat{j}$$

$$\vec{F}_{AD}=\left(\frac{Il}{2}\right)\hat{j}\times\left(B_0\right)\hat{k}$$

$$\vec{F}_{AD}=\left(\frac{IlB_0}{2}\right)\hat{i}$$

$\therefore$ $\vec{F}_{Net}=\vec{F}_{AB}+\vec{F}_{BC}+\vec{F}_{CD}+\vec{F}_{AD}$

$$\vec{F}_{Net}=\left(IlB_0\right)\hat{i}-\left(IlB_0\right)\hat{j}$$

$\therefore$ Magnitude of net force $(F_{Net}) = \left(IlB_0\sqrt{2}\right)N$

8.(B) (I) $x_1 = (u_{x1})t = (30)(2) = 60m$

$x_2 = (130) + (u_{x2})(t-1) = 130 + (-20)(1) = 110m$

$\therefore$ $\Delta x = 50m$

(I) $y_1 = u_{y1}t - \frac{1}{2}gt^2 = (30)(2) - \frac{1}{2}(10)(2)^2 = 40\,m$

$y_2 = 75 + u_{y2}(t-1) - \frac{1}{2}g(t-1)^2 = 75 + 20\times1 - \frac{1}{2}\times10\times(1)^2 = 90\,m$

$\therefore$ $\Delta y = 50\,m$

(III) $v_{x1} = 30\,m/s$

$v_{x2} = -20\,m/s$

$\therefore$ $v_{x1} - v_{x2} = 50\,m/s$

(IV) $v_{y1} = u_{y1} + a_y t = (30) + (-10)(2) = 10\,m/s$

$v_{y2} = u_{y2} + a_y(t-1) = 20 + (-10)(1) = 10\,m/s$

$\therefore$ $v_{y1} - v_{y2} = 0$

9.(A) In isobaric process, P = constant, so P-T graph will be a straight line parallel to T-axis.

P-V graph will also be a straight line parallel to V-axis.

$\therefore$ $\frac{V}{T} =$ constant, so V-T graph will be a straight line with some finite positive slope.

$PV = nRT$

$\Rightarrow$ $P = \frac{mRT}{MV} \Rightarrow P = \frac{\rho RT}{M}$ $\therefore$ $\frac{\rho RT}{M}$ = constant or ρT = constant

Rectangular hyperbolic relationship.

10.(C) (I) $\frac{\mu}{v} - \frac{1}{-u} = \frac{\mu-1}{-R}$

$\therefore$ $\frac{\mu}{v} = \frac{-1}{u} - \frac{\mu-1}{R}$

Therefore, v is always negative and so image of O is always virtual.

(II) $\frac{\mu}{v} - \frac{1}{+u} = \frac{\mu-1}{-R}$ or $\frac{\mu}{v} = \frac{1}{u} - \frac{\mu-1}{R}$

So, v may be positive or negative. Hence, image of O may be real or virtual.

(III) $\frac{\mu}{v}-\frac{1}{-u}=\frac{\mu-1}{R}\Rightarrow\frac{\mu}{v}=\frac{\mu-1}{R}-\frac{1}{u}$

Here again, v may be positive or negative. Hence image of O may be real or virtual.

(IV) $\frac{\mu}{v}-\frac{1}{u}=\frac{\mu-1}{R}\Rightarrow\frac{\mu}{v}=\frac{\mu-1}{R}+\frac{1}{u}$ $\therefore$ v is positive. Hence image of O is real.

Section – 3

1.(2) Energy of each photon of incident light $E=\frac{hc}{\lambda}$

Number of photons in 1 milliwatt source $N=\frac{P}{E}=\frac{P\lambda}{hc}$ (P = power of source)

Number of photoelectron released = 0.5% of N

Photo electric current = $0.5\%\times N\times1.6\times10^{-19}$

$$=\frac{0.5}{100}\times\frac{10^{-3}\times4965\times10^{-10}\times1.6\times10^{-19}}{6.62\times10^{-34}\times3\times10^{8}}=2\times10^{-6}\ A=2\mu A$$

2.(44) Here, atomic masses are given (not the nuclear masses), but still we can use them for calculating the mass defect because mass of electrons gets cancelled both sides. Thus, mass defect,

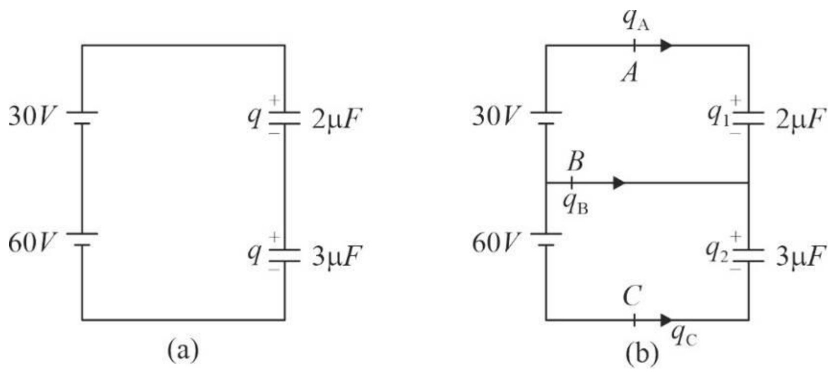
$$\Delta m=(22.9945-22.9898)$$

$$\Delta m=0.0047u$$

$$Q=(0.0047u)(931.5\,MeV/u)=4.4\,MeV$$

$\therefore$ $\frac{x}{10}=4.4$ $\Rightarrow$ $x=44$

3.(120) Let us draw two figures and find the charge on both the capacitors before closing the switch and after closing the switch.



Refer Figure (a), when switch is open both capacitors are in series. Hence, their equivalent capacitance is

$$C_{eq} = \frac{C_1C_2}{C_1 + C_2} = \frac{(2)(3)}{2+3} = \frac{6}{5}\mu F$$

Therefore, charge on both capacitors will be same. Hence, using $q = CV$, we get $q = (30+60)\left(\frac{6}{5}\right)\mu C = 108\mu C$

Refer Figure (b), when switch is closed, Let q_1 and q_2 be the charges (in μC) on two capacitors. Then, applying second law in upper and lower loops, we have

$30 - \frac{q_1}{2} = 0$ or $q_1 = 60\mu C$

$60 - \frac{q_2}{3} = 0$ or $q_2 = 180\mu C$

Charges q_1 and q_2 can be calculated alternatively by seeing that upper plate of $2\mu F$ capacitor is connected with positive terminal of 30 *V* battery. Therefore, they are at the same potential. Similarly, the lower plate of this capacitor is at the same potential as that of the negative terminal of 30*V* battery. So, we can say that *PD* across $2\mu F$ capacitor is also 30*V*.

$q_1 = (C)(PD) = (2)(30)\mu C$

$= 60\mu C$

Similarly, PD across $3\mu F$ capacitor is same as that between 60V battery.

Hence, $q_2 = (3)(60)\mu C$

$= 180\mu C$

Now, let q_A charge flows from *A* in the direction shown. This charge goes to the upper plate of $2\mu F$ capacitor. Initially, it had a charge +q and final charge on it is $+q_1$. Hence,

$q_1 = q + q_A$

Or $q_A = q_1 - q = 60 - 108$

$= -48\mu C$

Similarly, charge q_B goes to the upper plate of $3\mu F$ capacitor and lower plate of $2\mu F$ capacitor. Initially, both the plates had a charge $+q - q$ or zero. And finally they have a charge $(q_2 - q_1)$.

Hence, $(q_2 - q_1) = q_B + 0$

$q_B = q_2 - q_1 = 180 - 60$

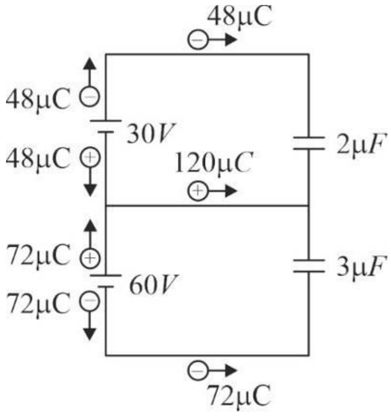
$= 120\,\mu C$

Charge q_C goes to the lower plate of $3\mu F$ capacitor. Initially, it had a charge $-q$ and finally $-q_2$.

Hence, $-q_2 = (-q) + q_C$

$$q_C = q - q_2 = 108 - 180 = -72\mu C$$

So, the charges will flow as shown below



4.(292) For the given condition, $u = -30cm,\ f = +20cm$

Using the lens formula, we have $\frac{1}{v} - \frac{1}{-30} = \frac{1}{+20}$

Solving this equation, we get $v = +60cm$ and $m = \frac{v}{u} = \frac{+60}{-30} = -2$

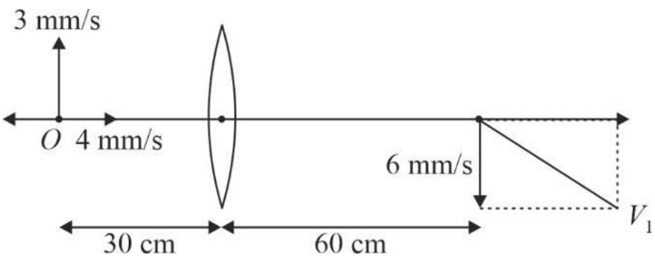
$m^2 = 4$

Component of velocity of object along the axis $= 5\cos 37° = 4mm/s$ (towards the lens)

Component of velocity of image along the axis $= m^2(4mm/s) = 4 \times 4 = 16\,mm/s$. This component is away from the lens (in the same direction of object velocity component).

Component of velocity of object perpendicular to the axis = $5\sin 37° = 3mm/s$ (upwards).

$\therefore$ Component of velocity of image perpendicular to axis $= m(3mm/s)$ or (-2) (3 mm/s) $= -6\,mm/s$ or this component is 6 mm/s downwards. These all points are shown in the figure given below.



$$v_I = \sqrt{(16)^2 + (6)^2} = \sqrt{292}\,mm/s$$

$$\tan\theta = \frac{6}{16} = \frac{3}{8}$$

$$\theta = \tan^{-1}(3/8)$$

5.(4) $R = 2v \times T = 2v\sqrt{\frac{2h}{g}}$

$h = 2r$

$R = 2v\sqrt{\frac{4r}{g}} \Rightarrow R = 4v\sqrt{\frac{r}{g}}$

$n = 4$

6.(17) $v_{COM} + R\omega = 2v$ **…(i)**

$v_{COM} - R\omega = -v$ **…(ii)**

From the above two equations we get $v_{COM} = \frac{v}{2}, \omega = \frac{3v}{2R}$

The kinetic energy in rolling is the sum of translational kinetic energy and rotational kinetic energy and is given by:

$$K = \frac{1}{2}Mv_{CM}^2 + \frac{1}{2}I_{CM}\omega^2$$

So, $K = \frac{1}{2}8m\left(\frac{v}{2}\right)^2 + \frac{1}{2}\left(\frac{8mR^2}{2}\right)\left(\frac{3v}{2R}\right)^2 = \frac{11}{2}mv^2$

$\Rightarrow$ Total Kinetic energy of the system $(K_T) = \frac{1}{2}m(2v)^2 + \frac{1}{2}(2m)v^2 + \frac{11}{2}mv^2$

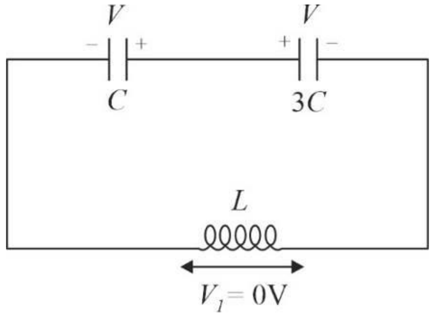
$\Rightarrow$ $K_T = \frac{17}{2}mv^2$

So $n = 17$

7.(3) When the current across inductor is maximum, $\frac{dI}{dt} = 0$

So potential difference across inductor when maximum current passes through it $(V_1) = -L\frac{dI}{dt} = 0V$

& the potential difference across both the capacitors at that time will be same (say V)



Situation when maximum current flows through inductor

Applying law of conservation of charges on plates of both the capacitors that are joined together by same wire.

Total initial charge = Total final charge

$\Rightarrow \quad (2V_0)(3C) - CV_0 = CV + 3CV \Rightarrow \quad 6CV_0 - CV_0 = 4CV \Rightarrow \quad V = \frac{5}{4}V_0$

Now loss in energy of capacitors by that time = Gain in energy of inductor by that time

$$\Rightarrow \quad \frac{1}{2}CV_0^2 + \frac{1}{2}(3C)(2V_0)^2 - \frac{1}{2}(4C)\left(\frac{5V_0}{4}\right)^2 = \frac{1}{2}LI^2$$

$$\Rightarrow \quad \frac{1}{2}(13CV_0^2) - \frac{1}{2}\left(\frac{25CV_0^2}{4}\right) = \frac{1}{2}LI^2 \Rightarrow \frac{1}{2}\left[\frac{52CV_0^2 - 25CV_0^2}{4}\right] = \frac{1}{2}LI^2$$

$$\Rightarrow \quad \frac{27CV_0^2}{4} = LI^2 \Rightarrow \quad I^2 = \frac{27CV_0^2}{4L} \Rightarrow \quad I = \frac{3V_0}{2}\sqrt{\frac{3C}{L}} \therefore \quad n = 3$$

8.(2) Let v be the speed of the projectile at highest point and $r_{\max}$ its distance from the centre of the earth. Applying conservation of angular momentum and mechanical energy,

$$mv_0 R_e \sin\alpha = mvr_{\max} \sin 90°$$

$$mv_0 R_e \sin\alpha = mvr_{\max} \quad \textbf{...(i)}$$

$$\frac{1}{2}mv_0^2 - \frac{GM_em}{R_e} = \frac{1}{2}mv^2 - \frac{GM_em}{r_{\max}} \quad \textbf{...(ii)}$$

Solving these two equations with the given data we get, $r_{\max} = \frac{3R_e}{2}$

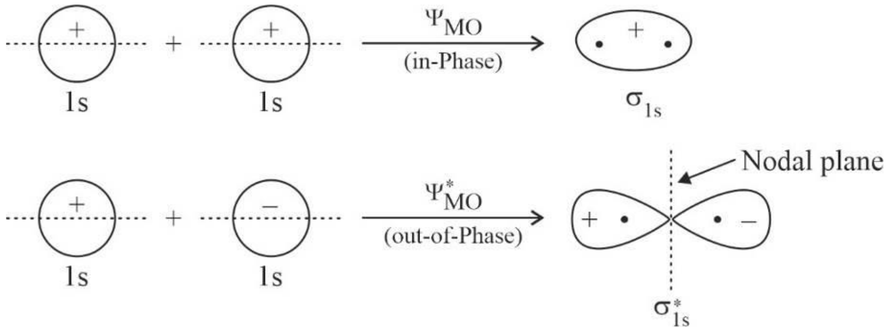
Or the maximum height $h_{\max} = r_{\max} - R_e = \frac{R_e}{2}$

So, $x = 2$

CHEMISTRY

Section – 1

1.(BD)



2.(CD) The enthalpy change during physisorption is in the range of 20 to 40 kJ mol^{-1}

Physisorption is an exothermic process

Physisorption results in multimolecular layer.

3.(AC) (A), (C) Heating of carbonates and hydroxide ores in absence of air to convert into their corresponding oxides is called calcination.

(B) Heating of sulphide in presence of air or O_2 to convert in to oxides is called roasting.

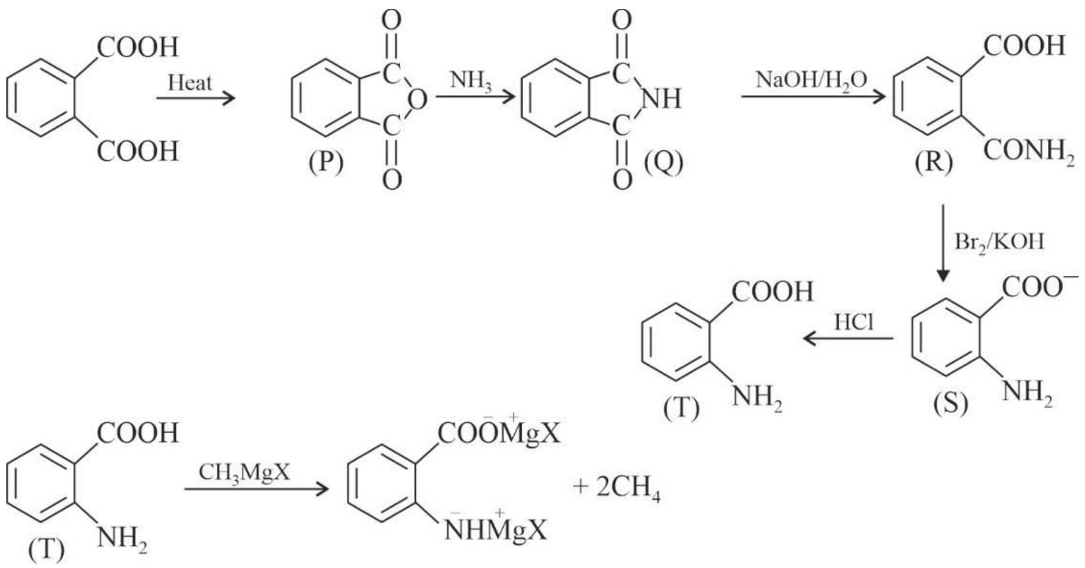
(D) This reaction represents the self-reduction.

4.(ABCD)

$$H_2S + 2HNO_3 \rightarrow 2NO_2 + 2H_2O + S$$

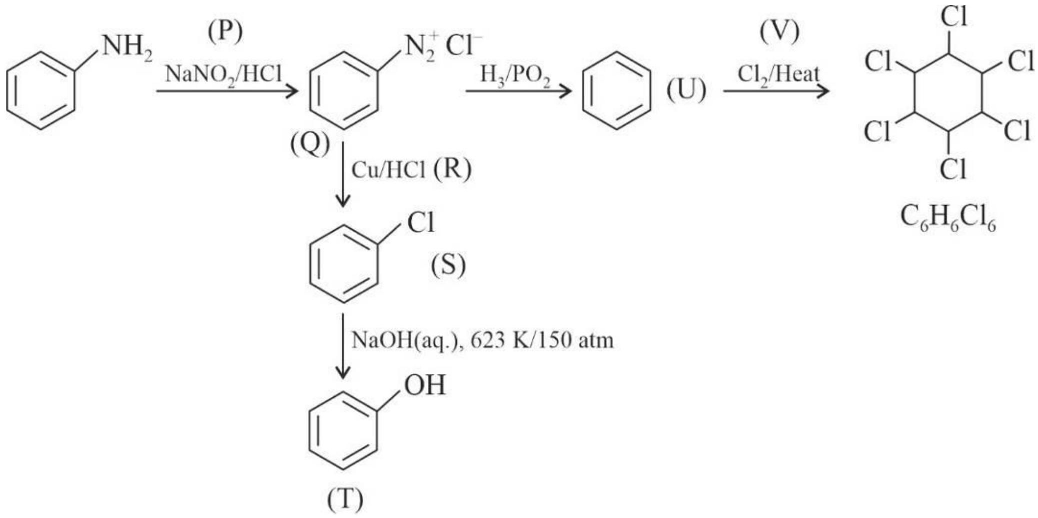
NO_2 is a paramagnetic, bent in geometry, an acidic oxide and reddish brown gas

5.(ABD)



P = Phthalic anhydride
Q = Phthalimide

6.(AC)



Section – 2

7.(B) I $\Rightarrow$ Q, R, S

II $\Rightarrow$ Q, R, S

III $\Rightarrow$ P, Q, R, S

IV $\Rightarrow$ P, R

(A) $r = k[A][B]$

Means 2nd order reaction

Unit of $K = mol^{-1}L^{1}S^{-1}$

$$-\frac{d[A]}{dt} = -\frac{d[B]}{dt} = k[A][B]$$

$$-\frac{d[A]}{dt} = \frac{d[C]}{dt}$$

$(t_{1/2})_A = (t_{1/2})_B$ If A and B are taken in stoichiometric ratio.

(B)

$A + B \rightarrow C + D$

$r = k_2[B]^0[A] = k_2[A]$

unit of $k = S^{-1}$

$$-\frac{d[A]}{dt} = -\frac{d[B]}{dt} = k_2[A]$$

$$-\frac{d[A]}{dt} = \frac{d[C]}{dt}$$

(C) $A + B \rightarrow C + D$

$$r = k_3[B]^0[A]^0 = k_3$$

$$\text{unit of } k = \text{mol}^1 L^{-1} S^{-1}$$

$$-\frac{d[A]}{dt} = -\frac{d[B]}{dt} = k_3$$

$$-\frac{d[A]}{dt} = \frac{d[C]}{dt}$$

(D) $2A + B \rightarrow 2C + 3D$

$$r = k_3[B]^0[A]^0 = k_3$$

$$\text{unit of } k = \text{mol}^1 L^{-1} S^{-1}$$

$$-\frac{1}{2}\frac{d[A]}{dt} = -\frac{1}{2}\frac{d[C]}{dt} = k_3$$

$$-\frac{d[A]}{dt} = \frac{d[C]}{dt}$$

8.(A)

I = P, Q, R

II = S

III = P

IV = P, R

Theory based

9.(A) I = P, Q, R

II = P, S

III = Q, R

IV = Q, R

(A) Ni^{2+} ions in both complexes have same primary valencies, i.e., + 2 same number of ions i.e. 3 and so have same conductance. Both have same EAN i.e. 34 Cl^- ions in both complexes are in coordination sphere so no precipitate is obtained with $AgNO_3$.

(B) III $\left[Co\left(NH_3\right)_6\right]Cl_3$ and IV $\left[Pt\left(NH_3\right)_5 Cl\right]Cl_3$ have different primary valencies but same number of ions. So same electrical conductance. Do not have same effective atomic number as in both complexes metals have different atomic number. As there are three Cl^- ions out side the coordination sphere both will gives 3 moles of the precipitate of AgCl (white).

(C) $\left[Pt\left(NH_3\right)_2 Cl_2\right]$ and $\left(NH_4\right)_2\left[PtCl_4\right]$. Pt is in +2 oxidation state so same primary valencies and same effective atomic number (84) and does not have same electrical conductance (former is neutral complex where as later one has 3 ions in aqueous solutions). No precipitate with $AgNO_3$ as Cl^- ions in both complexes are in coordination sphere.

(D) Both have Fe in +2 oxidation state so same primary valencies but have different number of ions, so different electrical conductance (former has 3 ions where as later one has 5 ions in aqueous solutions). Both complexes have same effective atomic number (i.e. 36).

10.(B) (i) Tollen's reagent $\left[AgNO_3 + NH_4OH \text{ or } \left\{Ag\left(NH_3\right)_2\right\}^+ OH^-\right]$:

Tollen's Reagent gives silver mirror or Black precipitate with aldehydes

$$R-CH=O \xrightarrow{AgNO_3+NH_4OH} R-\overset{\overset{\displaystyle O}{\|}}{C}-O^- + \underset{\text{silver mirror}}{Ag\downarrow}$$

Iodoform Test:

Reagents: $I_2 + NaOH$ or $NaOI$ (Where R = H, alkyl, aryl group)

Acetaldehyde, all methyl ketones, ethyl alcohol and all methyl alcohols give Iodoform test.

Lucas Reagent test (Conc. HCl + anhydrous $ZnCl_2$)

It gives white turbidity or cloudiness with alcohols (OH groups attached with sp^3 carbon).

Lucas Reagent used to distinguish between 1°, 2°, 3° alcohol because 1°, 2°, 3° alcohols react with different rate.

Phenols and enols do not give Lucas test.

Neutral $FeCl_3$ test:

It form coloured complex with phenol or enol (OH groups attached with sp^2 carbon)

$$6PhOH + FeCl_3 \longrightarrow \left[\left(PhO\right)_6 Fe\right]^{3-} + 3HCl + 3H^+$$

(Coloured complex)

It does not give test with alcohol.

2, 4-DNP = 2, 4 - Dinitrophenyl hydrazine Test :

Carbonyl compounds (all aldehydes and ketones) give yellow–orange precipitate with 2,4–DNP. It is also known as **Brady's reagent**.

Section – 3

1.(73) $\Delta T = 308 - 298 = 10K$

Molar heat capacity of calorimeter = 20 KJ K^{-1}

Heat released by combustion of 1 mole Zn(g) = - 20 x 10 = - 200 KJ

$$\Delta U_{Combustion} = -200 KJ\ mol^{-1}$$

$$Zn_{(g)} + \frac{1}{2}O_{2(g)} \rightarrow ZnO(s)$$

$\Delta n_g = 0-(1+0.5) = -1.5$

$\Delta H_{combustion} = \Delta U_{Combustion} + \Delta n_g RT$

$\Delta H_{combustion} = -200\times 10^3 + (-1.5)\times 8.3\times 298$

$\Delta H_{combustion} = -200\times 10^3 - 3710.1 = -200\times 10^3 - 3.710\times 10^3$

$\Delta H_{combustion} = -203.7\times 10^3 = -203.7 KJ\ mol^{-1}$

$Zn_{(s)} + \frac{1}{2}O_{2(g)} \rightarrow ZnO(s)$

$\Delta H^0_{combustion} = \Delta H^0_{f\,(ZnO)} - \Delta H^0_{f\,(Zn_{(s)}\rightarrow Zn_{(g)})} + \frac{1}{2}\Delta H^0_{f\,(O_2)}$

$-203.7 = \Delta H^0_f(ZnO) - 130.7 + 0$

$\Delta H^0_f(ZnO) = -203.7 + 130.7 = -73 KJ$

2.(905)

(i) $Cr^{2+}(aq.) + 2e^- \rightarrow Cr(s); \Delta G^0_1 = -2\times F\times E^0_1$

(ii) $Cr^{3+}(aq.) + 3e^- \rightarrow Cr(s); \Delta G^0_2 = -3\times F\times 0.74$

(iii) $Cr^{3+}(aq.) + e^- \rightarrow Cr^{2+}(s); \Delta G^0_3 = -1\times F\times 0.41$

Now,

$(i) = (ii) - (iii)$

$\Delta G^0_1 = \Delta G^0_2 - \Delta G^0_3$

$-2\times F\times E^0_1 = -3\times F\times 0.74 - (-1\times F\times 0.41)$

$2\times E^0_1 = 3\times 0.74 - 1\times 0.41$

$E^0_1 = \frac{2.22-0.41}{2}$

$E^0_1 = \frac{1.81}{2} = 0.905V = 905\ mV$

3.(11.96)

	H_2S +	KOH $\longrightarrow$	KHS +	H_2O
Initial mol	0.02	0.02	0.02	
After neutralisation	0	0	$0.02+0.02$ $=0.04$ mol	

Now, mixture contains 0.04 mole K_2S and 0.04 mole of KHS, so it is a Buffer. (HS^- / S^{-2})

$$pH = pKa_2 + \log\frac{[S^{-2}]}{[HS^-]}$$

$$pH = 11.96 + \log\frac{[0.04]}{[0.04]}$$

$$pH = 11.96 + \log 1$$

$$pH = 11.96 + 0$$

$$pH = 11.96$$

4.(0.95) Mole of $MnSO_4 = \frac{1.51}{151} = 0.01$ mole

$$MnSO_4 + 2Na_2CO_3 + 2KNO_3 \xrightarrow{\text{Heat}} \underset{\text{(green)}}{Na_2MnO_4} + 2KNO_2 + 2CO_2 + Na_2SO_4$$

0.01 mole 0.01 mole

$$3Na_2MnO_4 + 4H_2SO_4 \longrightarrow \underset{\text{(Pink)}}{2NaMnO_4} + 4NaHSO_4 + MnO_2 + 2H_2O$$

3 mole of Na_2MnO_4 gives 2 mole of $NaMnO_4$

0.01 mole ……………………… $\frac{2}{3} \times 0.01 = 0.0067$ mole

Mass of $NaMnO_4 = 0.0067 \times 141.92 = 0.95$ gm

5.(1.7) $CaO + 3C \xrightarrow{1000°C} \underset{(P)}{CaC_2} + \underset{(Q)}{CO}\uparrow$

2.8 gm

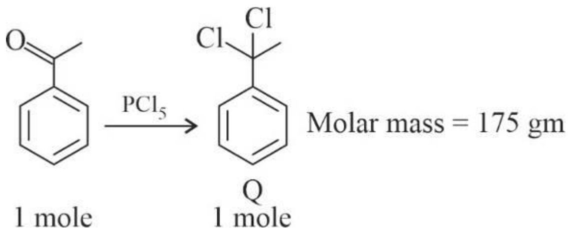
$$\underset{(P)}{CaC_2} \xrightarrow{N_2} \underset{(R)}{CaNCN} + C$$

$$\underset{(R)}{CaNCN} + 3H_2O \longrightarrow CaCO_3 + 2NH_3\uparrow$$

Mole of CaO $= \frac{2.8}{56}$

Mass of $NH_3 = \frac{2.8}{56} \times 34 = 1.7$ gm

6.(5.74)



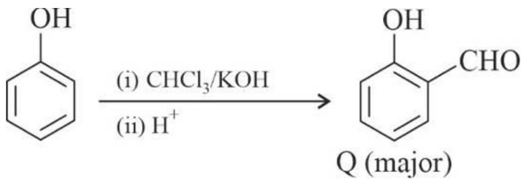
Molecular mass of $Q = 8 \times 12 + 1 \times 8 + 35.5 \times 2 = 175$ gm

Mole of $Q = \frac{3.5}{175} = 0.02$moles

Mole of $AgCl = 2 \times$ Mole of $Q = 2 \times 0.02 = 0.04$moles

Mass of $AgCl = 0.04 \times 143.5 = 5.74$ gm

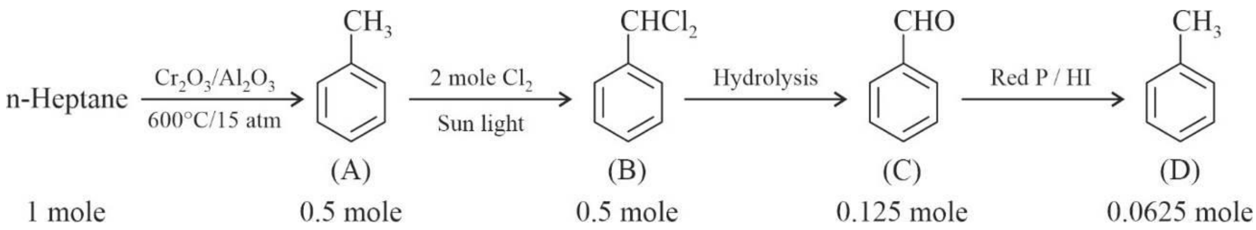
7.(26.23)



Molar mass of product $Q = 7 \times 12 + 1 \times 6 + 16 \times 2 = 122$ gm

Weight % of Oxygen $= \frac{32}{122} \times 100 = 26.23\%$

8.(5.75)



Molar mass of Toluene$(D) = 12 \times 7 + 1 \times 8 = 92$ gm

Mass of $D = 0.0625 \times 92 = 5.75$ gm

MATHEMATICS

Section – 1

1.(AB) $\left(\frac{x}{e}\right)^x = t, \log t = x(\log x - 1), \frac{1}{t}dt = \left((\log x - 1) + x\frac{1}{x}\right)dx$

$\Rightarrow \quad \frac{1}{t}dt = \log_e x\,dx \qquad I = \int_{1/e}^{1}\left(t^2 + \frac{1}{t}\right)\frac{dt}{t} = \int_{\frac{1}{e}}^{1}\left(t + \frac{1}{t^2}\right)dt = \left.\frac{t^2}{2} - \frac{1}{t}\right]_{1/e}^{1} = \frac{-1}{2} - \frac{1}{2e^2} + e$

2.(ABCD) $a_n(n+1) - n\,a_{n+1} = n^2(n+1) \Rightarrow \frac{a_n}{n} - \frac{a_{n+1}}{n+1} = n \Rightarrow a_1 - \frac{a_{n+1}}{n+1} = \frac{n(n+1)}{2}$

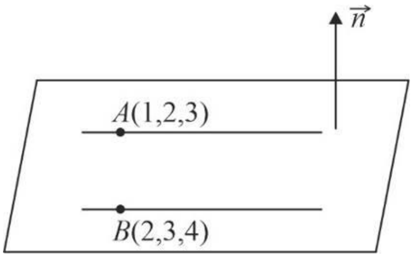
$\Rightarrow \quad 50 - n\left(\frac{n+1}{2}\right) = \frac{a_{n+1}}{n+1}$

3.(ABCD)

$$\begin{vmatrix} 2-1 & 3-2 & 4-3 \\ k & 2 & 3 \\ 2 & k & 3 \end{vmatrix} = 0$$

$\Rightarrow \quad (k-3)^2 = 1 \Rightarrow k = 2$ or 4

For $k = 2$, lines are parallel

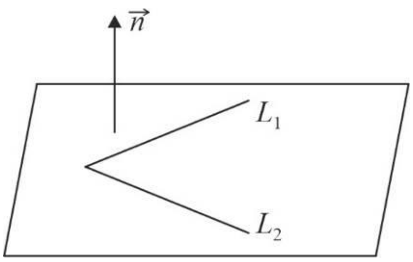


$\vec{n} = \overrightarrow{AB} \times \left(2\hat{i} + 2\hat{j} + 3\hat{k}\right) = \hat{i} - \hat{j}$

Plane: $(x-1) - (y-2) + 0(z-3) = 0$

$\Rightarrow \quad x - y + 1 = 0$

For $k = 4$, lines are intersecting

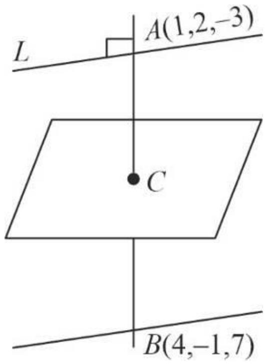


$\vec{n} = \vec{n}_1 \times \vec{n}_2 = -6\hat{i} - 6\hat{j} + 12\hat{k}$

Plane: $1(x-1)+1(y-2)-2(z-3)=0$

$x+y-2z+3=0$

4.(AB)



Mid-point $C\left(\frac{5}{2},\frac{1}{2},2\right)$ and normal has DR^s $(3,-3,10)$

Hence $3\left(x-\frac{5}{2}\right)-3\left(y-\frac{1}{2}\right)+10(z-2)=0$

$\Rightarrow \quad 3x-3y+10z-26=0$

$$LL' = 2\times\frac{|10(3)+1(-3)+(-6)(10)-26|}{\sqrt{(3)^2+(-3)^2+10^2}} = \sqrt{118}$$

5.(ABCD) Equation of tangents drawn to parabola $y^2 = 4x$ from point $P(-1,2)$ are given by $SS_1 = T^2$ i.e.,

$(y^2-4x)(y_1^2-4x_1) = (yy_1 - 2(x+x_1))^2$

$\Rightarrow \quad (y^2-4x)(4+4) = [2y-2(x-1)]^2 \quad \Rightarrow \quad 8y^2-32x = 4[y-x+1]^2$

$\Rightarrow \quad 2y^2-8x = (y^2+x^2+1-2xy-2x+2y)$

$\Rightarrow \quad y^2-x^2-6x+2xy-2y-1=0$ …(i)

Now, (i) would intersect line $x=2$ where

$y^2-4-12+4y-2y-1=0$

i.e., $y^2+2y-17=0$

$\Rightarrow \quad y_1+y_2=-2; y_1y_2=-17$

Where y_1 and y_2 are ordinates of points *A* and *B* respectively and $y_1 > y_2$

$\Rightarrow \quad AB = |y_1 - y_2| = \sqrt{(y_1+y_2)^2 - 4y_1y_2} = \sqrt{(-2)^2 - 4(-17)} = \sqrt{72} = 6\sqrt{2}$

$\therefore$ Area of $\Delta PAB = \frac{1}{2}(PM)\times(AB) = \frac{1}{2}(PK+KM)\times(6\sqrt{2})$

$= \frac{1}{2}(1+2)\times 6\sqrt{2} = 9\sqrt{2}$ square units

Also centroid of $\Delta PAB \equiv \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

$\equiv \left(\frac{-1+2+2}{3}, \frac{2-2}{3}\right) \equiv (1,0)$. That is, focus of parabola

6.(ABCD)

Let $x = \begin{vmatrix} 0 & \sin^2\theta - \tan^2\theta & \frac{1+\sin\theta}{\cos\theta} \\ \tan^2\theta\sin^2\theta & 0 & \cos^2\theta - 1 \\ \frac{\cos\theta}{\sin\theta - 1} & \tan^2\theta\cos^2\theta & 0 \end{vmatrix}$ is a skew symmetric matrix so $|x| = 0$

Now, $f(\theta) = \begin{vmatrix} 1+\sin^2\theta & \sin^2\theta & \sin^2\theta \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 2\theta & 4\sin 2\theta & 1+4\sin 2\theta \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$ and taken out $(2+4\sin 2\theta)$

$A = (2+4\sin 2\theta)$

$\begin{vmatrix} 1 & 1 & 1 \\ \cos^2\theta & 1+\cos^2\theta & \cos^2\theta \\ 4\sin 2\theta & 4\sin 2\theta & 1+4\sin 2\theta \end{vmatrix} = (2+4\sin 2\theta)\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & \cos^2\theta \\ -1 & -1 & 1+4\sin 2\theta \end{vmatrix} = (2+4\sin 2\theta)$

$f(\theta)_{\max} = 6$ at $\theta = \frac{\pi}{4}$

$2 + 4\sin 2x = 7$

$4\sin 2x = 5$

$\sin 2x = \frac{5}{4}$

$x =$ no solutions

Now $2 + 4\sin 2x = 4$

$4\sin 2x = 2$

$\sin 2x = \frac{1}{2}$

Section – 2

7.(A) (I) $e^{x}+e^{-x}=\tan x$

$e^{x}+\dfrac{1}{e^{x}}=\tan x$

$\Rightarrow$ $\tan x>2$ $\Rightarrow$ one solution in $\left[0,\dfrac{\pi}{2}\right)$

(II) $\cos x+\cos y=\dfrac{3}{2}$

$2\cos\left(\dfrac{x+y}{2}\right).\cos\left(\dfrac{x-y}{2}\right)=\dfrac{3}{2}$

$2\cos\dfrac{\pi}{3}.\cos\left(\dfrac{x-y}{2}\right)=\dfrac{3}{2}$

$\cos\left(\dfrac{x-y}{2}\right)=\dfrac{3}{2}$ $\Rightarrow$ No solution

(III) & (IV)

(III) $\cos x+2\sin x=1$

$\dfrac{\cos x}{\sqrt{5}}+\dfrac{2\sin x}{\sqrt{5}}=\dfrac{1}{\sqrt{5}}<1$

$\Rightarrow$ Two solution in $(0, 2\pi]$

(IV) $\sin x=\dfrac{\sqrt{3}}{2}, \cos x=\dfrac{1}{2}$ $\Rightarrow$ $x=2n\pi+\dfrac{\pi}{3}, n\in I$ and

$\sin x=-\dfrac{\sqrt{3}}{2}, \cos x=-\dfrac{1}{2}$ $\Rightarrow$ $x=2n\pi+\dfrac{4\pi}{3}, n\in I$

$\Rightarrow$ Infinite solutions.

8.(B) T.C = $3\times5\times7=105$

F.C for $P=3\times5\times4=60$

$\therefore$ $P(O)=\dfrac{60}{105}=\dfrac{4}{7}$ $\therefore$ $P(E)=1-\dfrac{4}{7}=\dfrac{3}{7}$

For x_1, x_2, x_3 in A.P.

$\Rightarrow$ $x_1+x_3=2x_2$ $\Rightarrow$ $x_1+x_3=$ even

$\Rightarrow$ $(1,1,1)(1,3,5)(5,3,1)(3,3,3)(3,5,7)(5,5,5)$ are only cases.

$$\therefore \qquad P\left(x_1, x_2, x_3 \text{ are in A.P}\right) = \frac{6}{105} = \frac{2}{35}$$

Now $P\left(\frac{\text{at least one of } x_1, x_2, x_3 \text{ is } 5}{x_1, x_2, x_3 \textit{ in } \text{A.P}}\right) = \frac{4}{6} = \frac{2}{3}$

9.(A) $\begin{vmatrix} a^2 & a^2-(b-c)^2 & bc \\ b^2 & b^2-(c-a)^2 & ca \\ c^2 & c^2-(a-b)^2 & ab \end{vmatrix}$ $\qquad C_2 \to C_2 - 2C_1 - 2C_3$

$$= -\left(a^2+b^2+c^2\right)\begin{vmatrix} a^2 & 1 & bc \\ b^2 & 1 & ca \\ c^2 & 1 & ab \end{vmatrix} = \left(a^2+b^2+c^2\right)(a-b)(b-c)(c-a)(a+b+c)$$

10.(C) Equation of tangents

$$y = -\frac{4}{3}x \pm 8 \Rightarrow 4x+3y = \pm 24$$

$$A(\pm 6, 0), B(0, \pm 8)$$

Section – 3

1.(1) $\alpha = 3\cos^{-1}\left(\frac{5}{\sqrt{28}}\right) + 3\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$\alpha = 3\tan^{-1}\left(\frac{\sqrt{3}}{5}\right) + 3\tan^{-1}\left(\frac{\sqrt{3}}{2}\right)$$

$$\alpha = 3\tan^{-1}\left(\frac{\frac{\sqrt{3}}{5}+\frac{\sqrt{3}}{2}}{1-\frac{3}{10}}\right)$$

$$\alpha = 3\tan^{-1}\left(\sqrt{3}\right) = \pi$$

$$\beta = 4\sin^{-1}\left(\frac{7\sqrt{2}}{10}\right) - 4\tan^{-1}\frac{3}{4}$$

$$\beta = 4\tan^{-1}(7) - 4\tan^{-1}\frac{3}{4}$$

$$\beta = 4\tan^{-1}(1) = \pi$$

$$\cos(\alpha-\beta) = \cos 0 = 1$$

2.(0) $\lim_{x\to a} f\left(g(x)\right) = f\left(\lim_{x\to a} g(x)\right)$ is possible if let at $L = \lim_{x\to a} g(x), f(x)$ is continuous at $x = a$, so

$$\lim_{x\to 2} \frac{\sin\left[\frac{e^{(x-2)}-1}{(x-2)}\times(x-2)\right]}{\frac{\log\left(1+(x-2)\right)}{(x-2)}(x-2)}$$

$$\lim_{x\to 2} \frac{\sin(x-2)}{x-2} = 1$$

So $\left[\tan\left(\frac{\pi}{12}\right)\right] = 0$

3.(5) Given $1-(1-P_1)(1-P_2)(1-P_3) = \frac{3}{4}$

$\Rightarrow \quad (1-P_1)(1-P_2)(1-P_3) = \frac{1}{4} \quad \ldots(1)$

$\Rightarrow \quad 1-(P_1+P_2+P_3)+(P_1P_2+P_2P_3+P_3P_1-P_1P_2P_3) = \frac{1}{4}$

$\Rightarrow \quad \sum P_1P_2 - 2P_1P_2P_3 = \frac{1}{2} \quad \ldots(2)$

Also, $P_1P_2(1-P_3)+P_2P_3(1-P_1)+P_3P_1(1-P_2) = \frac{2}{5}$

$\Rightarrow \quad \sum P_1P_2 - 3P_1P_2P_3 = \frac{2}{5} \quad \ldots(3)$

Solving (1), (2) and (3) we get

$P_1P_2P_3 = \frac{1}{10}, \sum P_1P_2 = \frac{7}{10}, \sum P_1 = \frac{27}{20}$

Now a cubic having roots P_1, P_2, P_3 is $x^3 - \sum P_1.x^2 + \sum P_1P_2.x - P_1P_2P_3 = 0$

i.e. $x^3 - \frac{27}{20}x^2 + \frac{7}{10}x - \frac{1}{10} = 0$

$\Rightarrow \quad 20x^3 - 27x^2 + 14x - 2 = 0$

4.(1) We have $\frac{1+z+z^2}{1-z+z^2} = 1+2\frac{z}{1-z+z^2} \in R$ if and only if $\frac{z}{1-z+z^2} \in R$.

That is, $\frac{1-z+z^2}{z} = \frac{1}{z} - 1 + z \in R$, i.e., $z + \frac{1}{z} \in R$

The last relation is equivalent to

$z+\frac{1}{z}=\bar{z}+\frac{1}{\bar{z}}$, i.e., $(z-\bar{z})\left(1-|z|^2\right)=0$

We find $z=\bar{z}$ or $|z|=1$

Because *z* is not a real number, it follows that $|z|=1$, as desired.

5.(1) $z^n(\lambda+i)=\bar{z}(-\lambda+i)$

Taking the absolute values of both sides of the equation, we obtain $|z|^n=|\bar{z}|=|z|$, hence $|z|=0$ or $|z|=1$.

If $|z|=0$, then $z=0$ which satisfies the equation. If $|z|=1$, then $\bar{z}=\frac{1}{z}$ and the equation may be rewritten as $z^{n+1}=\frac{-\lambda+i}{\lambda+i}$

Because $\left|\frac{-\lambda+i}{\lambda+i}\right|=1$, there exists $t\in[0,2\pi]$ such that $\frac{-\lambda+i}{\lambda+i}=\cos t+i\sin t$.

Then $z_k=\cos\frac{t+2k\pi}{n+1}+i\sin\frac{t+2k\pi}{n+1}$

For $k=0,1,......,n$ are the other solutions to the equation (besides *z* = 0), which lies on a circle of radius 1.

6.(34) $212(101+.......201)+222(101+.....201).....302(101+......201)$

$=(101+......201)\times(212+222+....302)=\frac{101}{2}[202+100]\times\frac{10}{2}[424+90]$

$=\frac{101\times302}{2}\times10\times\frac{514}{2}$

$101\times151\times5\times514=39195070$

7.(60) $n(x_1)=3, x_1=2,3,4$.

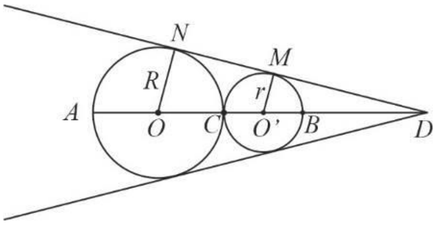
$4\le x_2<x_3\le8\Rightarrow$ from 4, 5, 6, 7, 8

Selecting any 2 $\Rightarrow$ $n(x_2,x_3)={}^5C_2=10$ and $n(x_4)=2$

$x_4=0,5$

$\Rightarrow$ Number of such 4 digit numbers $3\times10\times2=60$

8.(1)



Let $BD=x$

$r = \lambda x$

$R = 3\lambda x$

$$\frac{O'M}{ON} = \frac{O'D}{OD}$$

$$\frac{\lambda x}{3\lambda x} = \frac{\lambda x + x}{5\lambda x + x}$$

$\lambda = 1$